SUBBAND DECOMPOSITION AND FREQUENCY WARPING FOR SPECTRAL ESTIMATION

David Bonacci, Patrice Michel and Corinne Mailhes

ENSEEIHT/TéSA, National Polytechnic Institute of Toulouse
2 Rue Camichel, BP 7122, 31071 Toulouse Cedex 7, France
phone: +33 5 61 58 84 78, fax: +33 5 61 58 80 14, email: david.bonacci,patrice.michel,corinne.mailhes@tesa.prd.fr

ABSTRACT

Subband decomposition has already been shown to increase the performances of spectral estimation but induced frequency overlapping may be troublesome, bringing edge effects, when spectral estimation is applied after subband decomposition and decimation. This paper proposes a new spectral estimation procedure based on subband decomposition and frequency warping which reduces the overlapping frequency problem. Simulation results confirm the interest of this new algorithm.

1. INTRODUCTION

Subband decomposition has been used successfully in many signal and image processing applications such as speech, image and video compression [1], [2]. Some authors have recently shown that subband decomposition can also be a powerful tool for spectral estimation [3], [4]. In particular, the performance of traditional spectral estimation methods can increase when applied to signals filtered by an appropriate filterbank rather than applied to the corresponding fullband signal. This property has been theoretically explained as follows:

- for an AR(q) process, the minimum pth-order prediction error of the fullband exceeds the aggregate of the minimum pth-order prediction errors of the subbands, for $p \leq q$ [3],
- the local SNRs and frequency spacing increase by the decimation ratio [4].

These interesting results have been demonstrated for a bank of ideal infinitely sharp bandpass filters. Some experimental results have shown that these results can also apply to non-ideal filterbanks such as modified Quadrature-Mirror Filters (QMF's) [3] or cosine modulated filterbank [4]. However, when using such non-ideal bandpass filters, the same harmonic component can appear in two contiguous subbands at two different frequencies. This problem referred to as *spectral overlapping* occurs when the harmonic frequency is close to the subband edges. Of course, this spectral overlapping can be very troublesome in applications where the number of harmonic signals is unknown. The main contribution of this paper is to study a new spectral estimation procedure based on subband decomposition and frequency-warping which mitigates the spectral overlapping problem.
Section 2 presents the problem formulation and section 3 is devoted to the proposed method: the coupling of frequency warping and subband decomposition before spectral estimation. Simulation results are presented in section 4 and conclusions are reported in section 5.

2. PROBLEM FORMULATION

The observed signal is the sum of p sinusoids corrupted by additive white Gaussian noise:

$$x_t = \sum_{i=1}^{p} A_i \cos(2\pi f_i t + \phi_i) + n_t, \qquad (1)$$

where $t = 1, ..., N$. The problem of estimating the frequencies f_i from the observed samples x_t,$t = 1, ..., N$ has received considerable attention in the signal processing literature (see for instance [5] and references therein). As a consequence, many algorithms have been studied to solve this spectral estimation problem. These algorithms include nonlinear least squares (LS), High-Order Yule-Walker, Pisarenko and MUSIC methods [5]. Algorithms based on the Singular Value Decomposition (SVD) of the autocorrelation matrix have became very popular because of their high resolution properties and their insensitivity to model order overestimation. In this paper, we use the high-order Yule-Walker (HOYW) frequency estimation method ([5], p. 151) which is summarized below:

- estimate the $N \times N$ autocorrelation matrix of x_t denoted $\hat{R}_x$,
- compute the SVD of $\hat{R}_x$,

- solve the rank-truncated HOYW system of equations in the LS sense, which yields the estimated AR parameter vector denoted $\hat{a}$,
- determine the peaks of the pseudospectrum

  $$S(e^{j\omega}) = \frac{1}{|A(e^{j\omega})|^2},$$

  where $A(z)$ is the estimated AR polynomial (Z transform of the AR parameter estimates).

To illustrate the spectral overlapping problem, consider a pure sinusoidal signal embedded in additive white Gaussian noise:

$$x_t = A_1 \cos(2\pi f_1 t + \phi_1) + n_t, \tag{2}$$

where $A_1 = 1$, ϕ_1 is uniformly distributed on $[0, 2\pi[$ and the additive noise variance is $\sigma^2 = E[n^2(t)] = 0.5$ (the signal to noise ratio is SNR $= 10 \log\left(\frac{A_1^2}{2\sigma^2}\right) = 0dB$). The signal x_t is filtered by an 8-channel cosine modulated filter bank as in [4]. Figure 1 shows the averaged pseudospectra associated to the 5th and 6th subbands (corresponding to $[0.25, 0.3125[$ and $[0.3125, 0.375[$), computed from 50 Monte Carlo runs (the frequency is $f_1 = 0.31$). As can be seen, two different peaks appear in the two contiguous subbands, because the frequency of the sinusoidal signal $f_1 = 0.31$ is close to the subband edge.

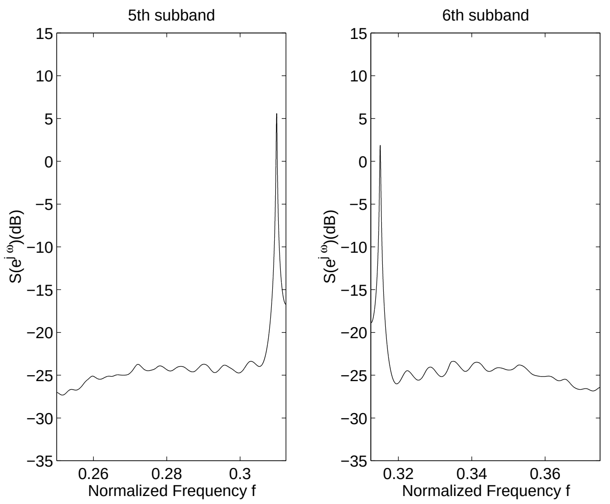

Figure 1: Average pseudospectra associated to the 5th and 6th subbands of a filtered pure sinusoidal signal.

3. DISCRETE FREQUENCY WARPING

This section studies a subband discrete frequency warping (SDFW), which allows to mitigate the spectral overlapping problem due to subband decomposition. The proposed SDFW consists in filtering the original signal (before subband decomposition) by a cascade of L causal first-order allpass filters with Z-transform

$$W(z) = \frac{z^{-1} - b}{1 - bz^{-1}}\ . \tag{3}$$

The corresponding structure and its equivalent schematic symbol are depicted in Figures ($3.a$) and ($3.b$). Straightforward computations show that $W(z)$ evaluated on the unit circle can be written $W(e^{j\omega}) = e^{-j\theta_b(\omega)}$, where

$$\theta_b(\omega) = \omega + 2 \arctan\left(\frac{b \sin\omega}{1 - b\cos\omega}\right), \tag{4}$$

which reveals the allpass structure of the filter $W(z)$. Note that the proposed SDFW is very similar to the frequency warped Laguerre filterbanks studied in [6] and shown in Figure 4. However, our approach does not require the time reversal operation. Moreover, the convolution with the zero-order Laguerre filter $\Lambda_0(z)$ has been removed from Evangelista's structure, which allows frequency warping without any amplitude distorsion. Note that the perfect reconstruction properties are not satisfied in absence of $\Lambda_0(z)$. However, such properties are not required for spectral estimation.

By denoting $U_m(z; b)$ the Z-transform of the mth allpass filter ouput, the following result can be easily obtained:

$$\begin{aligned} U_m(e^{j\omega}; b) &= Y(e^{j\omega}) W^m(e^{j\omega}) \\ &= Y(e^{j\omega}) e^{-jm\theta_b(\omega)}, \end{aligned} \tag{5}$$

where $Y(z)$ denotes the Z-transform of the input sequence y_t, $t \in \mathbb{Z}$. As a consequence, for a pure harmonic signal $x_t = A_1 \cos(2\pi f_1 t + \phi_1)$, the output of the mth allpass filter expresses as:

$$u_m(t; b) = A_1 \cos(2\pi f_1 t + \phi_1 - m\theta_b(\omega_1)) \tag{6}$$

where $\omega_1 = 2\pi f_1$. This expression for $u_m(t; b)$ is clearly periodic with respect to m, with period $\frac{2\pi}{\theta_b(\omega_1)}$. Consequently, $u_m(t; b)$ admits the following discrete-time Fourier (DTFS) series representation

$$u_m(t; b) = \sum_{m=-\infty}^{+\infty} c_m e^{jm\theta_b(\omega_1)}, \tag{7}$$

where the DFTS coefficients are clearly defined as follows

$$c_m = \begin{cases} \frac{A_1}{2} e^{j(2\pi f_1 t + \phi_1)} & m = 1 \\ \frac{A_1}{2} e^{-j(2\pi f_1 t + \phi_1)} & m = -1 \\ 0 & \text{else} \end{cases}$$

In other words, the frequency-domain representation of $u_m(t; b)$, $m \in \mathbb{Z}$, consists of two complex sinusoids with frequencies $\pm\frac{\theta_b(\omega_1)}{2\pi}$. This analysis shows that the filter-bank structure depicted in Figure 3(a) can be used as a DFW, which warps each frequency f_1 to $\frac{\theta_b(\omega_1)}{2\pi}$. The corresponding frequency warping depends on parameter b, as depicted in Figure 5. Figure 2 shows the 5th and 6th subband spectra for three different frequency warpings (i.e. three different values of parameter b). As can be seen, the spectral overlapping problem disappears for an appropriate value of b. More precisely, one peak appears in the 6th subband for $b = 0$ and disappears for $b = -0.094$ and $b = -0.183$.

Subband decomposition introduces aliasing side effects in the neighborhood of the cut-off frequencies of each filter. Then, the parameter b of the warping filters $W(z)$ has to be well-chosen: for example, if $b = -0.183$ causes a peak to disappear in the 6th subband, there is still a spurious peak in the 4th subband. Thus, it is of great importance to adjust frequency warping in order that each frequency bin is warped just in the middle of its corresponding frequency interval. Let us divide the frequency interval $[0, 0.5[$ into M equal subbands and note $f_j = (j-1)\frac{0.5}{M}$, $j = 1, ...M$ the cut-off frequencies. The algorithm used to reduce this aliasing effect can be summarized as follows:

for $j = 1$ to M

for $f = f_j$ to f_{j+1}

1 - warping parameter b selection: warp frequency f to $f_{j_m} = \frac{f_{j+1}-f_j}{2}$ using:

$$b(f) = \frac{1}{\cos(2\pi f) + \frac{\sin(2\pi f)}{\tan(\pi(f - f_{j_m}))}} \qquad (8)$$

2 - warping operation: apply the set of filters $W(z)$ to the original process x_t to get a warped signal u so that:

$$S_{x_t}(e^{j2\pi f}) = S_u(e^{j2\pi f_{j_m}}) \qquad (9)$$

3 - Subband decomposition: use the jth filter of the chosen filterbank (here cosine modulated filters, see [4]) on the warped signal u to get a filtered and decimated version y_j.

4 - Spectral estimation : perform the HOYW method described in section 2 on this subband signal to estimate uniquely

$$S_u(e^{j2\pi f_{j_m}}) = M^2 S_{y_j}(e^{j2\pi M f_{j_m}}), \qquad (10)$$

end for.

end for.

It should be noted that Eq. (10) is verified theoretically for ideal infinitely sharp bandpass filters. But, as the frequency under interest, f_{j_m}, lies in the middle of the j^{th} subband, the amplitude of all other filters can be assumed to be negligible. This ensures the validity of this equation.

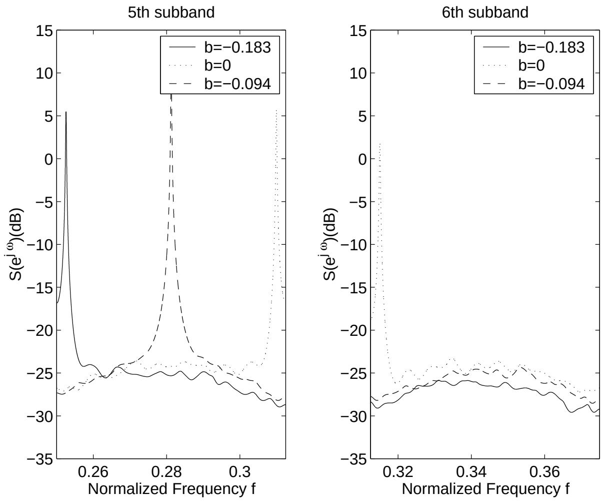

Figure 2: Average pseudospectra associated to the 5th and 6th subbands using frequency warping.

4. SIMULATION RESULTS AND PERFORMANCES OF THE METHOD

In order to highlight the performances of the proposed method SDFW, the power of the linear prediction error (LPE) is computed as a function of the analysed frequency. Let us consider the signal defined in Eq. (2). Different simulations will be carried on with a normalized frequency f_1 varying between 0 and 0.5. Using the exact autocorrelation of the signal:

$$r_x(\tau) = \frac{{A_1}^2}{2}\cos(2\pi f_1 \tau) + \sigma^2\delta(\tau), \qquad (11)$$

straightforward computations lead to the expression of the LPE power σ_e^2 as a function of the frequency f_1:

$$\sigma_e^2(f_1) = \frac{{A_1}^2}{2}\left[\frac{-2\alpha\cos^4(2\pi f_1) + \alpha\cos^2(2\pi f_1) + \beta}{\gamma - cos^2(2\pi f_1)}\right] \qquad (12)$$

where $\alpha = 2\mathrm{SNR}^{-1}$,$\beta = 2\mathrm{SNR}^{-1}+3\mathrm{SNR}^{-2}+\mathrm{SNR}^{-3}$,$\gamma = (1+\mathrm{SNR}^{-1})^2$ and $\mathrm{SNR} = \frac{{A_1}^2}{2\sigma^2}$.
When AR estimation is preceded by subband decomposition, the expression of the LPE is the same as in Eq. (12) replacing f_1 by Mf_1 and SNR by MSNR. Figure 6 presents the evolution of this LPE power versus the frequency under interest f_1. Both experimental and theoretical results show that, in this particular case of an AR(2), subband decomposition clearly reduces the LPE power but side effects (in the subband

edges) are proportionally bigger. The main interest of the proposed SDFW method is to bring back estimation of each frequency component in the middle of its corresponding subband. In this case, the LPE power is constant and equal to:

$$\sigma_e^2 = \frac{{A_1}^2}{2} \frac{\mathrm{SNR}_{sub}^{-1}(2 + 3\mathrm{SNR}_{sub}^{-1} + \mathrm{SNR}_{sub}^{-2})}{(1 + \mathrm{SNR}_{sub}^{-1})^2}, \qquad (13)$$

with $\mathrm{SNR}_{sub} = M\mathrm{SNR} = \frac{MA_1^2}{2\sigma^2}$. Figure 7 show theoretical and experimental LPE power using a biased estimator of the autocorrelation.

5. CONCLUSION

In this paper, we presented a new method, SDFW, and we studied its performances for reducing aliasing effects in the neighborhood of the filter's frontier frequencies and obtained some gain on linear prediction error criterion. Nowadays, this algorithm is only applicable to slowly time-varying signal but future works may lead towards an adaptive algorithm, applicable to non-stationary signals.

6. REFERENCES

[1] J. W. Woods, *Subband image coding.* Boston (MA): kluwer, 1994.

[2] A. Gersho and R. M. Gray, *Vector quantization and signal compression.* Boston (MA): kluwer, 1992.

[3] S. Rao and W. A. Pearlman, "Analysis of linear prediction, coding, and spectral estimation from subbands," *IEEE Trans. Inf. Theory*, vol. 42, pp. 1160–1178, July 1996.

[4] A. Tkacenko and P. P. Vaidyanathan, "Sinusoidal frequency estimation using filter banks," in *Proc. IEEE ICASSP-2001*, (Salt Lake City, Utah), pp. 825–828, May 2001.

[5] P. Stoica and R. Moses, *Introduction to spectral analysis.* Englewood Cliffs NJ: Prentice-Hall, 1997.

[6] G. Evangelista and S. Cavaliere, "Frequency-warped filter banks and wavelet transforms: a discrete-time approach via Laguerre expansion," *IEEE Trans. Signal Processing*, vol. 46, pp. 2638–2650, Oct. 1998.

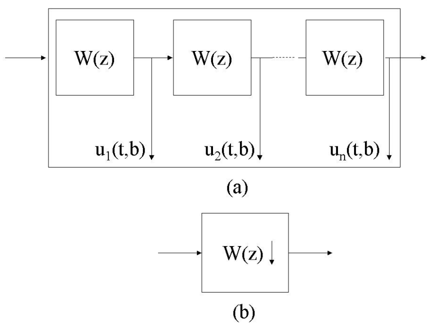

Figure 3: Filter cascade for frequency warping (a) and its symbolic representation (b).

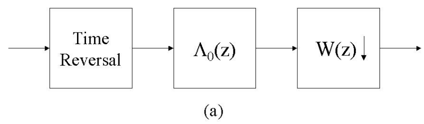

Figure 4: Frequency warped Laguerre filterbank.

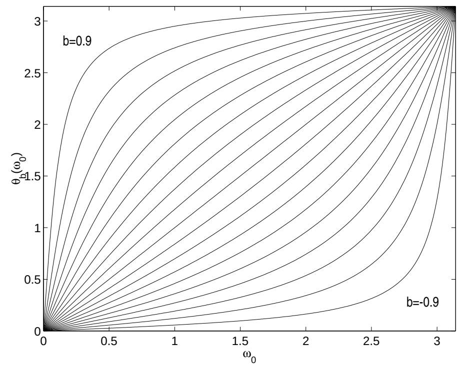

Figure 5: Warped frequency versus the original one for different values of parameter b.

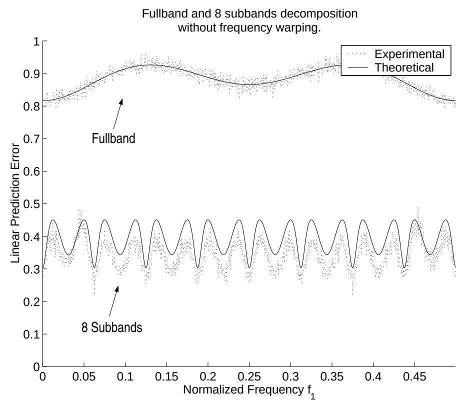

Figure 6: LPE power versus the analyzed frequency.

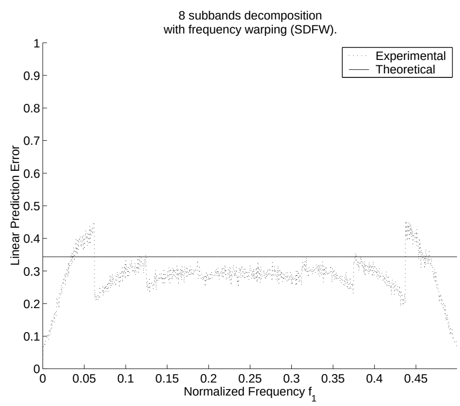

Figure 7: LPE power versus the analyzed frequency using frequency warping.